A THEORETICAL FRAMEWORK FOR EVALUATING DEBTOR-IN-POSSESSION FINANCING

Sandeep Dahiya∗
Korok Ray∗∗

ABSTRACT

The U.S. Bankruptcy Code provides enhanced priority and security features to debtor-in-possession (DIP) loans which can be obtained from a lender with whom the borrower may have no past lending relationship. The enhanced priority of DIP financing, and the choice of a DIP lender, significantly impact the investment decisions made by the firm. We show DIP loans from an existing lender leads to a higher level of investment. We also show that a higher priority of DIP financing also leads to higher investment by the firm. A bankruptcy judge should take these incentives into account when approving the DIP loan.

∗ Georgetown University – McDonough School of Business.
∗∗ Texas A & M University – Mays School of Business.

We are grateful to Eric Miller of CIT financial, Leonard Rosen of Wachtell, Lipton, Rosen and Katz, Larry King and William Allen of NYU Law School for providing us with the institutional details of DIP financing. We would like to thank Viral Acharya, Kose John, Loretta Mester, and Anthony Saunders for insightful comments and suggestions. Dahiya acknowledges the support of Stallkamp faculty fellowship grant provided by McDonough School of Business, Georgetown University. Please address all correspondence to Sandeep Dahiya, Rafik B. Hariri Building, McDonough School of Business, Georgetown University, Washington, D.C. 20057. Tel: (202) 687 3808; Fax: (202) 687 4031. Email: sd@georgetown.edu.
Philadelphia Newspapers LLC is accusing its pre-bankruptcy lenders of engineering a financing pact that hijacks the publisher’s restructuring process and places it in the hands of the lenders . . . the publisher of the Philadelphia Inquirer warned that its control over the [c]hapter 11 proceedings could be in jeopardy if its bankruptcy loan of choice is not approved . . . . The terms of the prepetition lenders’ DIP (debtor-in-possession) loan give those lenders a veto power . . . . The company remains locked in a dispute with the lenders over two bankruptcy loan proposals currently on the table. Philadelphia Newspapers prefers the $15 million package offered by Republic First Bank, but lenders say it strips them of necessary protections. They argue that their $15 million financing package is superior. Six hours of supervised mediation on Thursday was not enough to bring the two parties to a consensus . . . . As a result, the courtroom showdown over the bankruptcy loan has been postponed until next Friday, when a judge will also examine the “exclusivity” clause currently shielding the company from rival plans.1

—Dow Jones Institutional News, August 21, 2009

[Referring to auto industry bailout] . . . . The government functioned as a debtor-in-possession, or DIP, lender. DIP lenders take equity positions and negotiate a reorganization plan that makes sure that every single creditor is made better off than they would have been . . . .2

—Lawrence Summers, the director of the White House’s National Economic Council, Interview with Wall Street Journal, December 27, 2009

INTRODUCTION

When a firm can no longer meet its debt obligations should it be liquidated or should it be restructured? In a world with no frictions or asymmetric information the answer is straightforward: the choice that leads to preservation of most value is the socially optimal choice.3 However, significant uncertainty about the future prospects as well as conflict of interests between debt holders, equity holders, and operating managers can make this a difficult decision.4 A

---


3 How to divide the value of a firm that is unable to service its financial obligations between its various claim holders is an important, but separate question that we do not focus on in this paper. See generally Lucian A. Bebchuk, A New Approach to Corporate Reorganizations, 101 HARV. L. REV. 775 (1988) (providing a good review of optimal bankruptcy design).

4 See generally id.
primary public policy goal of the 1978 Bankruptcy Reform Act (commonly referred to as the Bankruptcy Code or simply as the Code) is to identify distressed firms, where the going-concern value is higher than the liquidation value. The goal is then to protect those firms against efforts to force their liquidation. The Code provides two key mechanisms to avoid immediate liquidation of a firm that defaults on its debt. First is the “automatic stay” provision and the second is “exclusivity period.” The automatic stay (§ 362 of the Code) ensures that a firm filing for reorganization under chapter 11 of the Code is protected from any civil actions (e.g., seizing collateral, collection of claims, creation of liens, etc.). The exclusivity period, provided under § 1121(b), ensures that the management team of the filing firm is the only party allowed to propose the initial plan of reorganization (POR). The combined effect of these two provisions is that the debtor (i.e., the incumbent management) retains control of the assets and operations. Thus, after filing for petition under chapter 11, the debtor is referred to as “debtor-in-possession” (DIP).

However, simply stopping the creditors from enforcing their collection claims is not enough to keep a firm operating as a going concern. A bankruptcy filing is likely to trigger a liquidity crisis for the firm. For example, most firms depend on credit from their suppliers to keep day-to-day operations running. Typically, few suppliers are willing to ship goods on credit to a firm operating under chapter 11, since their claim will not be secured, and will rank lowest in priority (trade credit is typically unsecured and receives the lowest priority under the Code). A chapter 11 filing would induce most suppliers to demand cash upfront, which can paralyze the smooth operations of the firm filing for chapter 11 protection. If the filing firm can line up a new source of

---

6 See id. (“The purpose of a business reorganization, unlike a liquidation case, is to restructure a business’s finances so that it may continue to provide its employees with jobs, pay its creditors, and produce a return for its shareholders.”).
8 Id. § 1121(b).
9 See id. § 362.
10 Id.
13 See Skeel Jr., supra note 11, at 1930.
15 See 7 COLLIER ON BANKRUPTCY ¶ 1129.03[3] (Alan N. Resnick & Henry J. Sommer eds., 16th ed.).
financing (e.g., a new line of credit), it may be able to overcome such liquidity problems.\textsuperscript{17} Potential lenders, however, are unlikely to provide new debt to a borrower that has filed for bankruptcy.\textsuperscript{18} This reluctance arises from significant uncertainty about the repayment ability of the borrower.\textsuperscript{19} Thus, there are few funding options for a firm that is operating under the protection of chapter 11. The Code addresses this problem by providing special creditor rights to loans made after the chapter 11 filing.\textsuperscript{20} Specifically, § 364 of the Code allows for a special kind of post-petition financing, usually referred to as DIP financing.\textsuperscript{21} The DIP lenders enjoy certain rights, which are unavailable to the creditors of firms not operating under chapter 11 protection.\textsuperscript{22} Historically, the U.S. bankruptcy law has always allowed some form of special financing for distressed firms (especially railroads) attempting to reorganize their operations.\textsuperscript{23} However, the 1978 Code went much farther in providing broader rights to DIP financing.\textsuperscript{24} DIP financing has become an increasingly common source of financing for U.S. firms filing for protection under chapter 11.\textsuperscript{25} Dahiya et al. report that the fraction of bankrupt firms, who obtain DIP financing, has been rising.\textsuperscript{26} By the mid-1990s, almost half of the public firms filing for chapter 11 obtained DIP financing.\textsuperscript{27} Ayotte and Morrison study chapter 11 filings by public firms in 2001 and report that 50 percent of the filers obtained DIP financing and an additional 26 percent obtained permission to use cash under terms very similar to those of DIP loans.\textsuperscript{28} DIP financing is also credited with making the reorganization process more creditor friendly as DIP lenders have used this type of lending to exert control over the


\textsuperscript{18} Mark L. Prager, \textit{Financing the Chapter 11 Debtor: The Lenders’ Perspective}, 45 BUS. LAW. 2127, 2127 (1990).


\textsuperscript{21} Id.


\textsuperscript{23} See Skeel, Jr., supra note 11, at 1908–13.

\textsuperscript{24} In the late nineteenth century, financially distressed railroads, attempting reorganization, would issue a “receiver’s certificate,” which was a promissory note that would have the highest priority. \textit{See id.} (providing a detailed discussion of the historical development of DIP financing).

\textsuperscript{25} Dahiya, supra note 22, at 260.

\textsuperscript{26} Id.

\textsuperscript{27} Kenneth M. Ayotte & Edward R. Morrison, \textit{Creditor Control and Conflict in Chapter 11}, 1 J. LEGAL ANALYSIS 511, 515 (2009).

\textsuperscript{28} Id. at 523.
For example, Bharath et al. and Adler et al. document significant reduction in absolute priority deviations in favor of equity holders in recent years. They argue that increased use of DIP financing is a key driver for their findings. Ayotte and Skeel propose that many features of the bankruptcy law, including provision for DIP financing, mitigate the liquidity problem of the borrower as few (if any) lenders will be willing to provide debt to a distressed borrower. The superior creditor control rights have made DIP financing as contract of choice for large scale restructuring involving private firms and government. For example, the U.S. Government chose to inject over $300 million in General Motors via a DIP line of credit. Similarly, to help with its restructuring, the city of Detroit used a $120 million DIP loan from Barclays bank, the first deal of its kind for a municipal bankruptcy.

Legal scholars, however, are not unanimous in endorsing DIP financing. It is possible that DIP financing can distort a firm’s investment choices. Bebchuk and Fried as well as Warren outline the drawbacks of senior and secured financing. Since DIP financing is an extreme form of senior secured lending, their argument against such loans is especially relevant. They argue that such credit provides strong risk-shifting incentives, specifically for shareholder aligned managers of a distressed firm. By pledging unencumbered assets as security, a borrower can transfer wealth from pre-existing unsecured creditors to the new secured creditors.

---

31 Bharath, supra note 30, at 11–12; Adler, supra note 29, at 464.
33 Skeel, supra note 11, at 1919–20.
37 Id. at 515.
40 See Bebchuk, supra note 38, at 873–74; Warren, supra note 38, at 1379.
41 Bebchuk, supra note 38, at 891–94.
financing is likely to worsen the wealth-transfer incentives of the management.\textsuperscript{42} Stultz and Johnson and Schwartz argue, however, that higher priority financing can address the under-investment problem better.\textsuperscript{43} The Code provides the bankruptcy judge with a wide latitude to approve an appropriate level of priority that a DIP loan can hold.\textsuperscript{44} Thus, it is useful to examine how the priority level of DIP loans affects the investment decision of the borrower. This paper aims to provide a theoretical framework, which allows a bankruptcy judge to evaluate the appropriate priority level of a DIP loan.

Another related issue for the bankruptcy court arises when a borrower requests a DIP loan from a lender who may or may not have any prior (pre-petition) loan exposure to that borrower. The Code allows a distressed borrower to get its DIP financing from either its existing (pre-petition) lender, or from a new lender.\textsuperscript{45} There are significant benefits for a pre-petition lender to continue as the DIP lender. For example, an existing lender already has a relationship with the borrower which provides the lender an information and cost advantage over a new lender.\textsuperscript{46} Furthermore, by not providing DIP financing, the existing lender risks diluting its security on the pre-petition loans.\textsuperscript{47} Finally, the Code allows (although rarely) for a pre-petitioned unsecured lender to provide DIP financing, and the use of collateral, to secure both the unsecured pre-petition loan as well as the post-petition loan.\textsuperscript{48} This rare type of DIP financing is allowed under the cross-collateralization clause of the Code.\textsuperscript{49}

These arguments imply that the pre-petition lead lender is the ‘‘Natural DIP Lender.’’\textsuperscript{50} There are, however, competing reasons that favor a new lender as the sole provider of DIP financing. First, there is considerable disagreement among the various claimholders regarding the value of the firm, and how it

\textsuperscript{42} Cf. id. at 870–71; Warren, supra note 38, at 1374–76.
\textsuperscript{45} Id. § 364(d).
\textsuperscript{47} See PAUL ZUMBRO, DIP AND EXIT FINANCING TRENDS AND STRATEGIES IN A CHANGING MARKET 6 (2016).
\textsuperscript{49} Id.
\textsuperscript{50} See Petersen, supra note 46, at 5–6; ZUMBRO, supra note 47, at 6–7.
should be divided among the various classes. The pre-petition lender (hereafter, the inside bank) enjoys substantial inside information compared to the trade creditor or public debtholder. The inside bank and the firm have strong incentives to collude and extract higher concessions from the uninformed claimholders. LoPucki and Whitford describe occurrences of this form of collusion, prior to the adoption of the absolute priority rule in 1939. Bulow and Shoven and Gertner and Scharfstein discuss the coalition formation by the shareholders and the inside bank to achieve similar wealth transfers. Berlin, John, and Saunders provide a theoretical setting, which describes a rational junior claimholder who anticipates the possible collusion of the inside bank and the firm. Therefore, such a creditor will be unwilling to accept the plan of reorganization proposed by this coalition. By allowing a new lender to provide DIP financing, a firm adds credibility to its reorganization plan. Also, the under-investment problem, caused by the debt overhang, may be mitigated if the new lender provides DIP financing.

Second, Sharpe and Rajan provide another reason for turning to a new lender for post-petition financing. Both studies show theoretical models of the bank-borrower relationship. These models show that the inside bank can

54 Id.
57 A number of recent news stories highlight these conflicts. For example, in the 2013 bankruptcy filing of GMX resources, both the unsecured creditors as well as a group of preferred shareholders filed objections to its $50 million DIP loan. See, e.g., Matt Chiappardi, GMX Investors Blast $50M Ch. 11 Loan as Unfair, LAW 360 (Apr. 26, 2013), https://www.law360.com/articles/436469/gmx-investors-blast-50m-ch-11-loan-as-unfair.
58 Darla D. Moore, How to Finance a Debtor in Possession, 6 COM. LENDING REV. 3, 8 (1990).
61 Sharpe, supra note 60, at 1071; Rajan, supra note 60, at 1370–72.
extract monopoly rents from its borrowers, due to its information advantage.\textsuperscript{62} This monopoly power likely plays a significant role in the period leading up to the chapter 11 filing, as the distressed debtor tries to renegotiate his debt.\textsuperscript{63} The increasing rent extraction by the inside bank can make the securing of DIP financing from a new lender more attractive.\textsuperscript{64} The assumption of a competitive market for the supply of loans implies there are many new lenders willing to extend DIP loans.\textsuperscript{65} In some instances, lenders use DIP financing as an opportunity to replace old banking relationships.\textsuperscript{66} These arguments imply that there are strong incentives for a firm to approach a new lender to provide post-petition credit.\textsuperscript{67}

Our discussion thus far, underscores the importance of priority level of DIP loans, and the choice of DIP lenders in shaping the investment decisions of borrowers. In this paper we provide a theoretical model that examines these issues. The two papers most closely related to our papers are Gertner and Scharfstein, and Triantis.\textsuperscript{68} Gertner and Scharfstein primarily focus on how priority and maturity structure of existing debt affects the restructuring outcomes.\textsuperscript{69} Our paper focuses more narrowly on the issues that arise due to special protection offered to DIP loans by § 364 of the Code. Triantis develops a theoretical model for judicial oversight of DIP financing.\textsuperscript{70} Our paper presents a more formal treatment of his model and provides extensions to the cases where a DIP loan enjoys “cross-collateralization” as well as analyzing the case of DIP financing being provided by an existing junior creditor. The financing by a junior creditor typically happens when a large trade creditor agrees to provide funding for restructuring. For example, Pinnacle Air, a small regional airline, obtained a $75 million DIP loan from Delta Airlines which was its largest trade creditor and customer at the time of chapter 11 filing.\textsuperscript{71}

\begin{thebibliography}{9}
\bibitem{62}Sharpe, supra note 60, at 1069.
\bibitem{63}Rajan, supra note 60, at 1368.
\bibitem{64}Sharpe, supra note 60, at 1069.
\bibitem{65}ZUMBO, supra note 47, at 15–16.
\bibitem{66}See e.g., David Neustadt, Lending to Bankrupt Companies Seen as Opportunity to Gain Some Assets, AM. BANKER, Oct. 27, 1987. Chemical Bank, the industry pioneer in 1990’s, used DIP financing to generate new lending and investment banking businesses from the firms that it previously had no relationship with. One of its managing directors stated, “we approach the market as a new business opportunity and typically have no existing exposure to the debtor.” Moore, supra note 58, at 8.
\bibitem{67}Neustadt, supra note 66; Moore, supra note 66, at 8.
\bibitem{68}Gertner, supra note 55, at 1189; Triantis, supra note 20, at 901.
\bibitem{69}Gertner, supra note 55, at 1189, 1192.
\bibitem{70}Triantis, supra note 20, at 918–19.
\end{thebibliography}
The plan of the paper is as follows: in Section 2 we briefly describe the chapter 11 process, and discuss the various financing options available to the firm operating under chapter 11. This section also provides the details of § 364 of the Code, which governs the DIP lending process. In Section 3, the basic model is laid out and solved for alternative bankruptcy regimes. We conclude in Section 4.

I. BACKGROUND

A. The Chapter 11 Reorganization Process

One of the major differences between state and federal non-bankruptcy debt collection laws, and the Code, is the idea of a fresh start for the debtors. For a corporate borrower, this is usually achieved via a chapter 11 filing. As described by Epstein, Nickles, and White, there are various stages to a chapter 11 case. A voluntary filing by the debtor marks the first stage of the reorganization process. The filing immediately results in an automatic stay on all payments to pre-petition creditors, all collection efforts, and all foreclosure actions. Thus, the automatic stay provides a major incentive for a distressed borrower to seek protection under chapter 11.

Simultaneously, this borrower reduces the power of pre-petition creditors, while allowing the incumbent management to maintain operating control of the debtor’s assets. The result is that the debtor is in possession of his own affairs or a “debtor-in-possession.” But the protection from creditors comes at a cost. The debtor-in-possession (i.e., incumbent management) faces several restraints on how the debtor-in-possession can use the estate’s assets, and, in

---

73 The use of chapter 11 is not restricted to a business entity only; some of the lower courts barred individual chapter 11 filings until the Supreme Court ruled that individual debtors may file for relief under chapter 11. Toibb v. Radloff, 501 U.S. 157, 166 (1991).
75 In few cases, the commencement of chapter 11 is initiated by an involuntary petition by the creditors under §303. 11 U.S.C. §§ 301, 303 (2012).
76 Id. § 362.
78 While the Code allows the bankruptcy judge to replace the existing management by a court appointed trustee to operate the firm, in practice, the appointment of a trustee is very rare, and incumbent management usually continues in place, at least in the period immediately after the filing. Id.
79 Id. at 452–53.
particular, the liquid assets, such as cash. The various guidelines and restrictions on the use, sale, and lease of assets are contained in § 363 of the Code.

The debtor-in-possession plays two critical roles. First, the debtor-in-possession formulates a plan of reorganization and, second, runs the day-to-day business of the firm. Once the firm has filed for bankruptcy, the incumbent management continues to operate its business while formulating a plan of reorganization. At this stage the firm requires some form of financing, including DIP financing. Next, we discuss the various financing options available to a firm operating under chapter 11.

B. The Post-petition Financing Process

Let us now turn to the institutional details of the post-petition financing process. Table 1 shows the menu of financing choices available to a firm after filing for chapter 11. The first and natural source of funds for a business is the cash flow generated by its operations. This can be a substantial resource, as the firm is no longer paying interest on its pre-petition debt. Although § 363 of the Code allows the firms to engage in the ordinary course of business without prior approval of the bankruptcy court, the use of cash is subject to extensive restraints. Section 363(c)(2) specifically prohibits even the ordinary use of “cash collateral” without permission from the court. Thus, obtaining court approval for the use of cash collateral provides the firm with its first source of liquidity.

81 Id.
82 This exclusivity period is for 120 days from the date of a chapter 11 filing but is usually extended. Thomas G. Kelch, The Phantom Fiduciary: The Debtor in Possession in Chapter 11, 38 WAYNE L. REV. 1323, 1327, 1330 (1992).
83 Id.
84 Moore, supra note 58, at 4–5.
86 Id.
87 Id.
The use of cash collateral may not adequately meet all the firm’s financing needs. Therefore, the firm may need to raise funds through DIP financing. In most cases, the firm tries to arrange DIP credit before the formal filing of the chapter 11 petition. If the firm is able to arrange DIP financing, then the petition is accompanied by a request of approval for DIP financing. DIP financing can be obtained under § 364 of the Code, which has four subsections. As Table 1 describes, “§ 364 was structured with an escalating series of inducements that a debtor-in-possession may offer to attract credit in the post-petition period.” Subsections (a) through (d) provide an increasing level of priority and security for the DIP lender. Section 364 is progressive in nature, since the benefits provided in each subsection are not available until it has been established that a good faith effort to obtain credit was unsuccessful.

DIP financing under § 364(a) is the easiest to arrange, as it does not require any court approval. It allows the debtor to obtain unsecured credit in the ordinary course of business. This credit must fund an expense that is otherwise eligible for treatment as an administrative expense under § 503(b), and enjoys the administrative expense priority. The restriction of credit under § 364(a) to the ordinary course of business means that this financing is usually limited to trade credit.

DIP financing under § 364(b) can be used for purposes other than the ordinary course of business, but its use must be approved by the bankruptcy court after a due notice and hearing. The lenders prefer this form of financing because it removes the ambiguity surrounding whether or not the

---

91 Id.
92 Id.
93 Wigness, supra note 85.
96 Id.
98 Id.
99 Id.
100 Within the class of general unsecured loans, §507(a) assigns different priorities to different types of unsecured claims. 11 U.S.C. § 507(a) (2012). Administrative expenses, as defined in §503(b), are entitled to first priority, among unsecured claims. 11 U.S.C. § 503(b) (2012). The examples of administrative claims include professional fees, costs of selling or liquidating assets, etc.
102 Id.
credit is made in the ordinary course of business. As with the credit under § 364(a), the loans under § 364(b) are unsecured.

Section 364(c) provides stronger incentives to the lenders. Under three clauses, it empowers the debtor to grant a DIP lender:

1. priority over all other administrative expenses in the case,
2. security interest in unencumbered assets, or
3. a junior lien on already encumbered assets.

Section 364(c), like § 364(b), requires court approval after the due notice and hearing. Furthermore, the debtor must prove to the court that it could not obtain financing on an unsecured basis under § 364(a) and § 364(b).

The court can approve financing under § 364(d) if the priorities and security offered by § 364(c) are insufficient to obtain new credit. This allows the debtor to offer a lien on the already pledged collateral that is senior to existing liens, referred to as priming liens. The approval of such a priming lien is subject to several requirements. First, as in § 364(c), the debtor-in-possession must prove that it was unable to obtain financing under § 364(a), § 364(b), or § 364(c). Second, the debtor-in-possession must prove that the interests of the lender being primed are adequately protected. Finally, § 364(e) protects the DIP lenders from the adverse effects of a subsequent reversal, or modification, on appeal of the bankruptcy court’s orders authorizing the super-priority and priming lien.

There is case law in which the pre-petition, unsecured lender agrees to provide DIP loans, provided that the collateral for the post-petition loan can also be pledged as security for the pre-petition, unsecured loans. This arrangement is called “cross-collateralization,” and provides additional incentives for a pre-petition lender to extend post-petition financing.

---

105 Id. § 364(c).
106 Id.
107 Id. § 364(d)(1).
108 Id. § 364(d)(1)(A).
109 See Epstein, supra note 103.
111 Id. § 364(d)(1)(B).
112 Id. § 364(e); Epstein, supra note 103.
114 Id.
Whether or not it is permitted under the Code remains a controversial and unsettled issue.\textsuperscript{115}

Table 2 provides the priority structure of claims for a hypothetical firm that has contracted post-petition financing under all the subsections of § 364.

II. \textsc{The Model}

The focus of this paper is to examine how DIP financing affects the investment incentives of a borrower. For a financially distressed firm, the legal recourse to the same, or higher priority, DIP financing is unique to the Code.\textsuperscript{116} We examine how a financially distressed firm would invest if the only source of DIP financing is from one of its existing borrowers. Here, we focus on how changing the priority of the DIP loan, compared to existing pre-petition loans, would influence the investment decisions of the borrower. Next, we introduce the possibility of obtaining DIP financing from a new lender, and examine its impact on the investment policy of the borrower.

Our model is similar to the one used by Gertner and Scharfstein.\textsuperscript{117} The model has two dates. At time 0, the only asset that the firm possesses is cash \( L \) and an opportunity to invest in a project requiring total outlay of \( I \). The firm has debt of face value \( B \), due at date 0 that is held by a syndicate of lenders. The firm is insolvent at date 0, i.e., \( B > L \). The investment opportunity can only be exploited by the firm and cannot be sold separately. Thus, the firm needs \( I - L \) to finance the investment opportunity, and it has the choice of arranging DIP financing from either its old lenders, or from a new lender. If no DIP loan can be arranged, the firm will be liquidated for \( L \). The project requiring an investment of \( I \), generates a stochastic cash flow \( X \), which is distributed over \([0, \infty)\), with a probability distribution \( f(X) \). All parties are assumed to be risk neutral, and the risk-free rate is assumed to be zero.\textsuperscript{118} Thus, in case the firm has a terminal cash flow \( (X) \), larger than the


\textsuperscript{116} For example, the U.K. insolvency law requires that an Administrator, typically a qualified insolvency practitioner, replaces the board of directors as the manager of the company in administration (i.e., reorganization). However, in recent years some countries are amending their bankruptcy statutes to allow some form of senior secured lending for the reorganizing firm. For example, in 2009 Canada amended its Companies Creditors Arrangement Act (CCAA) to codify the circumstances in which courts will allow DIP loans. See Epstein, supra note 103.

\textsuperscript{117} See Gertner, supra note 55.

\textsuperscript{118} While assumption of zero risk free rate is not realistic, it allows us to abstract away from determining the interest rate and, instead, allows us to focus on the effect that priority has on the investment incentives of
face value of the total debt ($B$), the residual surplus is retained by the coalition of DIP lender and the shareholders of the firm. This surplus can be shared between the borrower and lender, although we do not model the sharing contract. To keep model tractable, we make the same assumption as Gertner and Scharfstein and assign the entire surplus to the DIP lender.¹¹⁹

We define the first best investment policy (socially optimal) for the firm as one that increases the total value of the firm. Let $NPV^*$ (Socially Optimal) denote the minimum level of expected NPV of a project, such that the project would increase the value of the firm. It is easy to see that $(NPV)^*_{SociallyOptimal} = 0$, i.e., all positive NPV projects should be undertaken. Thus, every project requiring an investment $I$ that has an expected return, $\bar{X}$ greater than $I$, should be undertaken. In the next section, we examine how the presence of pre-existing risky debt, and the priority of DIP loans, affects the investment policy.

A. DIP Financing Provided by Pre-petition (Old) Lender

We shall first restrict our analysis where one of the firm’s existing lenders, which we refer to as “Old or Pre-petition Lender,” provides DIP loans. Additionally, we focus on the impact of different priority levels of DIP loans on the investment incentives of the borrower.

1. The Pre-petition Bank Provides Financing at the Same Priority as Pre-petition Loans

Assume that the bank provides the DIP loan and has a fraction $\phi$ of the pre-petition loan. Thus, the face value of its existing debt is $\phi B$. If the firm does not get DIP financing, it will be liquidated. In that case, the potential DIP lender with fraction $\phi$ of the old loan would realize a cash flow equal to $\phi L$. Thus, the pre-petition bank will provide a DIP loan only if:

$$\int_{0}^{\phi B + I - L} \frac{\phi B}{Z} f(X) dX + \int_{\phi B}^{\phi B + I - L} \left[ X - (1 - \phi)B \right] f(X) dX - (I - L) > \phi L. \quad (1)$$

Where $Z = B + (I - L)$ denotes the total face value of debt (DIP loan and pre-petition) at time 1. Equation 1 is the incentive compatibility constraint for the old lender willing to provide a DIP loan. The first term is the pay-off to the

¹¹⁹ See id.
DIP lender when the firm’s cash flow is not sufficient to cover the entire debt repayments. The second term is the pay-off to DIP lender when the cash flow is adequate to pay off the debt, and the DIP lender takes the residual value.\textsuperscript{120} The first term in Equation 1 can be rewritten as 
\[
\frac{z}{Z}B + I - L - \int_{0}^{z} \frac{1 - \varphi}{Z} Xf(X)dX
\]
Also the second term can be expanded and rewritten as 
\[
\int_{z}^{\infty} [Xf(X)dX - \int_{z}^{\infty} (1 - \varphi)Bf(X)dX]
\]
Finally, 
\[
\bar{X} = \int_{0}^{z} Xf(X)dX
\]
Thus, equation 1 can be rearranged as:
\[
\bar{X} - I > \frac{z}{Z}B + I - L + \int_{0}^{z} (1 - \varphi)Bf(X)dX
\]
Let us define \( V_{\varphi}^{\delta} \) as:
\[
V_{\varphi}^{\delta} = \frac{z}{Z}B + I - L + \int_{0}^{z} (1 - \varphi)Bf(X)dX
\]
Conceptually \( V_{\varphi}^{\delta} \) is the value of existing debt conditional on the firm obtaining a DIP loan regardless of who provides the financing. Substituting the value of \( V_{\varphi}^{\delta} \) in Equation 2, it can be rewritten as:
\[
\bar{X} - I > (1 - \varphi)[V_{\varphi}^{\delta} - L] = (NPV)_{\text{EqualPriority}}^{\delta}
\]
The expected NPV of undertaking the investment is \( \bar{X} - I \). Thus, the right hand side of the inequality in Equation 4 is the minimum expected NPV of the project, for which an old lender would provide DIP financing at a priority level equal to that of pre-petition loans. If \( [V_{\varphi}^{\delta} - L] \) is equal to zero, then we obtain the socially optimal outcome: all projects with positive NPV are financed. This is the case if the decision to obtain DIP financing makes the value of existing debt exactly equal to the pay-off if the firm is liquidated. This implies that there is no wealth transfer to or from existing debt holders arising from the decision to invest in the project financed by new DIP loan. This socially optimal outcome can also occur if the pre-petition lender is the only existing lender, i.e., \( \varphi = 1 \). In this case, the investment problem is trivial, since old lenders would always finance only positive NPV projects. If \( [V_{\varphi}^{\delta} - L] \) is not equal to zero, however, then the firm’s investment policy will deviate from the

\textsuperscript{120} For simplicity, we assume that entire surplus is paid to the DIP lender. This assumption can be relaxed to allow sharing of the surplus between the DIP lender and shareholders, but would add complexity without any additional insights on how priority of DIP loan affects the investment choice.
socially optimal. If \( V^d_s - L < 0 \), the right hand side of Equation 4 will be negative. Thus, the firm will invest in some projects that are negative NPV implying over-investment. On the other hand, if \( V^d_s - L > 0 \), the minimum cut-off point above which the firm will invest is strictly greater than zero. This implies that the firm will forego some positive NPV projects resulting in under-investment.

2. The Pre-petition Bank Provides DIP Financing at a Priority Higher to Pre-petition Loans

If the DIP lender is able to get higher priority, then the incentive compatibility constraint can be rewritten as:

\[
\int_s^1 Xf(X)dX + \int_{I-L}^s [(I-L)+\phi(X-(I-L))]f(X)dX;
\]

\[
+ \int_s^\infty [X-(1-\phi)B]f(X)dX - (I-L) > \phi L.
\]

(5)

The first term in Equation 5 is the pay-off to the DIP lender if cash flow is less than the face value of the DIP loan \((I-L)\). The second and third terms describe the pay-offs to DIP lenders at increasing levels of realized cash flows. The second term can be expanded and rewritten as:

\[
\int_s^1 (1-\phi)(I-L)f(X)dX + \int_{I-L}^s Xf'(X)dX - \int_{I-L}^\infty (1-\phi)Xf'(X)dX.
\]

Rearranging the terms gives:

\[
X-I > [(1-\phi)B]/X + \int_{I-L}^s (1-\phi)f(X)dX - \int_{I-L}^\infty (1-\phi)(I-L)f(X)dX - (1-\phi)L.
\]

(6)

Let us define \( V^H_s \) as:

\[
V^H_s = \int_{I-L}^\infty [X-(I-L)]f(X)dX + \int_{I-L}^\infty Bf'(X)dX.
\]

(7)

Conceptually, \( V^H_s \) is the value of an existing loan conditional on a firm obtaining a new DIP loan (regardless of who provides this loan) at a higher priority compared to existing loan. Substituting \( V^H_s \) in Equation 6 yields:

\[
X-I > (1-\phi)(V^H_s - L) = (NPV)_{\text{HigherPriority}}.
\]

(8)

Denoting \((1-\phi)(V^H_s - L)\) by \((NPV)_{\text{HigherPriority}}\), Equation 8 describes the minimum level of expected NPV from a project for which the pre-petition lender would provide DIP loans. The higher priority of DIP loan would induce the lender to finance a larger set of projects, since the minimum
acceptable NPV is lower for a lender, who gets higher priority for his DIP loan as summarized in the following lemma:

**Lemma 1 (L:L1) (NPV)_{equalPrio} is strictly greater than (NPV)_{higherPrio}.**

This implies that the cut-off level for NPV below which the project will not be financed is lower if the DIP loan is at a higher priority compared to the existing loans. Thus, a higher priority of a DIP loan results in a higher level of investment.

Intuitively, the increased willingness to invest, due to a higher priority of a DIP loan, addresses the well-known problem of “debt overhang” as discussed in Myers.\(^{121}\) Thus, for firms that have an attractive set of investment projects, but face a large debt overhang, access to a higher priority DIP loan would lead to more efficient investment. However, the higher priority may not be an optimal outcome for all borrowers. For borrowers with poor investment opportunities, higher investment may lead to inefficient risk-shifting in negative NPV projects.\(^ {122}\) For such borrowers, DIP financing should be provided (if at all) at equal priority.\(^ {123}\) We summarize this intuition in the following proposition:

**Proposition 1 (P:OldPriority)**

DIP financing at higher priority should only be granted for the borrowers that have good investment opportunities.\(^ {124}\)

3. **Cross-collateralization**

Cross-collateralization is a controversial feature of the Code, under which the bankruptcy judge can allow the old lender, providing DIP financing, to have higher priority for both the DIP financing and its proportion of the old loan.\(^ {125}\) In some cases, the pre-petition lender can “roll-up” its old loans into the new DIP loan, effectively using part of the

---

\(^{121}\) Stewart Myers, Determinants of Corporate Borrowing, 5 J. Fin. Econ. 147, 147–75 (1977).


\(^{123}\) Id.

\(^{124}\) Id.

\(^{125}\) Cross-collateralization has been defined by the Second Circuit as follows: “In return for making new loans to a debtor-in-possession under [chapter 11], a financing institution obtains a security interest on all assets of the debtor, both those existing at the date of the order, and those created in the course of the [chapter 11] proceeding.” This is not only for the new loans (the propriety of which is not contested), but also for existing indebtedness to it. Otte v. Mfrs. Hanover Com. Corp., 596 F.2d 1092, 1094 (2d Cir. 1979).
DIP loan to retire the old loan.\textsuperscript{126} Conceptually, a roll-up facility is similar to a cross-collateralized DIP loan.\textsuperscript{127} We examine the investment incentives if the DIP loan is cross-collateralized as highest priority.

The face value of total loans extended by a pre-petition DIP lender would equal $\phi B + I - L$, which can be rewritten as $Z - (1 - \phi)B = T$. In order for the DIP lender to be willing to provide financing that is cross-collateralized, the equation must hold:

$$\int X f(X) dX + \int \left[ X - (I + \phi B - L) \right] f(X) dX - (1 - \phi)L > \phi L. \tag{9}$$

The first term of Equation 9 reflects the cross-collateralization, since both the pre-petition loan exposure of the DIP lender as well as the new DIP loans enjoy first priority. Note that the fraction of pre-petition loan owned by lender(s) not providing the DIP loan is effectively subordinated. Rearranging the terms we get:

$$\overline{X} - I > \int X f(X) dX + \int \left[ X - (I + \phi B - L) \right] f(X) dX - (1 - \phi)L,$$

which can be rewritten as:

$$\overline{X} - I > (1 - \phi) \left( \int X f(X) dX + \int \left[ X - (I + \phi B - L) \right] f(X) dX - L \right). \tag{10}$$

Let us define $V^c$ as:

$$V^c = \int X f(X) dX + \int \left[ X - (I + \phi B - L) \right] f(X) dX. \tag{11}$$

Substituting $V^c$ in Equation 10 we get:

$$\overline{X} - I > (1 - \phi)[V^c - L]. \tag{12}$$

Thus $(1 - \phi)[V^c - L]$ is the minimum level of expected NPV for a project that would be financed by a cross-collateralized DIP loan. We denote this by $NPV^{\phi}$ (cross-collateral). Effectively, cross-collateralization is an even higher level of priority, compared to the case where only the DIP loan had higher priority. This can be summarized in the following proposition:

Proposition 2 (P:Cross)

For borrowers that have an attractive investment opportunity set, DIP financing can be cross-collateralized with the pre-petition debt. Conversely, for borrowers with a poor investment opportunity, set DIP financing should not be cross-collateralized.

Again, the result is expected due to an increase on the priority of the loan, which makes the lender willing to finance a larger set of projects.

So far, we have examined how the priority of a DIP loan affects the investment decisions of the firm. However, we restricted the firm’s choice of DIP lender to one of its existing pre-petition lenders. In the next section, we examine what happens when the firm raises DIP loans from a new lender. This new loan, the so called “pure debtor-in-possession financing,” happens when a lender with no pre-petition relationship to the firm agrees to provide DIP financing.128

4. New DIP Lender is Allowed

In this section, we show that for same level of priority, the minimum level of expected NPV, required to induce a new DIP lender to lend, is always higher than the minimum NPV required by a DIP lender, who also holds some pre-petition debt. This situation is captured in the following proposition:

Proposition 3 (P:NewLender1)

For the same level of priority, the minimum acceptable NPV for a project required by a new DIP lender is higher than the one required by the loan provided by a pre-petition lender.

The intuition here is that new DIP lender must break even on the DIP loan only as it has no outstanding debt. An existing borrower will be willing to finance a project with somewhat lower NPV as long it leads to some increase in value of existing loan. Thus, the existing lender may be willing to invest, even if the DIP loan does not break even on a stand-alone basis, as long as the old loan recovery level is high enough.

---

5. Same Priority DIP Financing from an Existing Lender vs. Higher Priority DIP Financing from a New Lender

Another interesting issue is when a bankruptcy judge is presented with competing offers from an existing lender, who is willing to provide DIP loans at equal priority, and a new lender, who is willing to provide DIP financing at a higher priority. While at first glance it may appear unfair to grant the new lender higher priority, the investment decision would in fact depend on how large a fraction of the pre-petition loan (i.e., $\phi$) is owned by the existing lender, who willingly provides DIP financing. This situation is formalized in the following proposition:

**Proposition 4 (P:NewLender2)**

DIP financing, from an existing lender at the same priority, would still lead to more investment than DIP financing at a higher priority from a new lender, only if the pre-petition lender holds a fraction $\phi > \phi_{\text{max}}$, where:

$$\phi_{\text{max}} = \frac{V^s_B - V^n_B}{V^s_B - L}.$$

This proposition captures the idea that if the old lender holds a large enough fraction of existing debt, the ability to increase the value of this outstanding debt is strong enough to induce the lender to provide DIP financing at the same priority and achieve similar investment incentives, which a new DIP lender will achieve only if granted higher priority.

B. DIP Financing from the Pre-petition Senior Lender vs. the Pre-petition Junior Lender

So far, we have considered a firm with only one class of pre-petition debt. In this section we examine what happens if the firm has two different classes of pre-petition debt and the DIP financing can be provided by either of them. While the basic framework of our model remains the same, the firm now has senior debt of face value $B$ and junior debt of face value $J$. We assume that that there are only two pre-petition lenders with one holding the senior debt and the other holding the junior debt. We also assume that the liquidation value $L$ of the firm is lower than $B$, so that both classes of creditors would lose part of the face value in case the firm is liquidated. The DIP financing is assumed to be at the same level as the
If the senior creditor provides the DIP financing, the following condition must be satisfied:

\[
\int_{0}^{\infty} X f(x) \, dx + \int_{0}^{z_1}(I-L+B) f(x) \, dx + \int_{z_1}^{\infty}(X-J) f(x) \, dx - (I-L) > L \implies \\
X - I > \int_{0}^{z_1} [X - (I-L+B)] f(x) \, dx + \int_{z_1}^{\infty} J f(x) \, dx = k_1,
\]

where \(Z_1 = I - L + B\) and \(Z_2 = I - L + B + J\).

The junior creditor would be willing to provide DIP financing if:

\[
\int_{0}^{\infty} \frac{I - L}{Z_1} X f(x) \, dx + \int_{0}^{z_1}(X - B) f(x) \, dx - (I-L) > 0 \implies \\
X - I > \int_{0}^{z_1} \frac{B}{Z_1} X f(x) \, dx + \int_{z_1}^{\infty} B f(x) \, dx - L = k_2.
\]

The right-hand side of Equation 13 gives the lowest NPV project that the firm with DIP financing from its senior creditors would be able to finance. Similarly, the right-hand side of Equation 14 gives the lowest NPV project for the firm with DIP financing from its junior creditor.

To examine the investment policy of the firm with DIP financing from different classes of pre-petition lenders, we compare \(k_1\) and \(k_2\). If \(k_2 - k_1 < 0\), the financing from junior creditor will lead to a greater investment as the minimum cut-off level of NPV is lower in that case. From Equations 13 and 14,

\[
k_2 - k_1 = \int_{0}^{z_1} \frac{B}{Z_1} X f(x) \, dx + \int_{0}^{z_1} B f(x) \, dx + \int_{z_1}^{\infty} B f(x) \, dx - L \\
- \int_{0}^{z_1} (X - Z_1) f(x) \, dx - \int_{z_1}^{\infty} J f(x) \, dx.
\]

The above equation can be rewritten as

\[
k_2 - k_1 = \int_{0}^{z_1} \frac{B}{Z_1} X f(x) \, dx + \int_{0}^{z_1} (2B + 1 - L - X) f(x) \, dx + \int_{z_1}^{\infty} (B - J) f(x) \, dx - L
\]

It is difficult to get sharp predictions on the investment policy of the firm from Equation 16. We restrict our analysis to the special case in which \(X\) is distributed uniformly over an interval \([0,M]\), where \(M > Z_1\). We further
assume that $M > 2I$, this implies that the firm invests only in positive NPV projects.

Let the total indebtedness of the firm be denoted by $T = B + J$. Thus, the combination of senior and junior debt can be denoted by $(B, B - T)$. The following proposition describes the effect of the mix of total debt $T$ on the investment incentives of the firm:

**Proposition 5 (P:5)** For any given level of total debt $T$, and for every level of senior debt $B$, there exists a unique level of liquidation $L^*_T(B)$ such that:

1. If the liquidation value of the firm $L > L^*_T(B)$, financing from the junior pre-petition lender would lead to higher levels of investment $(k_s - k_i < 0)$,

2. If the liquidation value of the firm $L < L^*_T(B)$, financing from the senior pre-petition lender would lead to higher levels of investment $(k_s - k_i > 0)$, where:

$$L^*_T(B) = \frac{B\left[\frac{1 + B}{2M} + (T - B)\left(\frac{2I + T + B}{2M}\right)\right]}{\left(1 - B^{2M}\right) + \left(\frac{T - B}{M}\right)}.$$  \hspace{1cm} (17)

**Conclusion**

The Code provides unusual incentives to potential lenders to provide credit to firms reorganizing under chapter 11. These include higher priority and enhanced security for the loans commonly referred to as DIP financing. 131 Furthermore, either the existing lenders, or a new lender, can provide the DIP financing. 132 A major requirement for DIP loans is the approval from the bankruptcy court. 133 This article focuses on the implications of allowing different priority levels of DIP loans, as well as the effect of allowing pre-petition versus new DIP lender to provide DIP loans.

---

132 See id.
133 See id.
Our main result is that a higher level of priority for DIP loans would provide incentives to invest a larger amount. This outcome would be desirable for firms with attractive investment opportunities but with large debt overhang. Thus, for firms faced with an “under-investment” problem, a bankruptcy judge should allow a high priority level for the DIP financing. The increased investment may not be optimal for firms that do not have many good investment opportunities. In this case, the high priority DIP financing can lead to dissipative investment in high risk, negative NPV projects.

A bankruptcy judge should keep this risk-shifting incentive in mind when approving DIP financing and should scrutinize the higher priority DIP financing proposals more closely. Cross-collateralization is shown to be an extreme form of high priority DIP financing and, as such, should be allowed only when there is a reasonable degree of certainty that the firm possesses good investment opportunities. In choosing between an existing or new lender to provide DIP financing, the bankruptcy judge would need to ascertain the desirability of higher investments because, for any given level of priority, DIP loans from existing lenders are likely to induce more investments. Thus, for firms faced with the “under-investment” problem, DIP loans from existing lenders would be optimal. For firms that are prone to risk-shifting, the higher profitability requirement of a new DIP lender would provide better monitoring.
APPENDIX

Proof of Lemma 1:

Equations 4 and 8 describe the minimum acceptable NPV for equal priority and higher priority DIP loans respectively. To prove Lemma 1, we need to show $V_h - V_h^* > 0$. Let $V_h - V_h^* = K$.

Substituting the values from Equations 3 and 7, we get:

$$K = \left( \int \frac{B}{Z} Xf(X) dX + \frac{B}{Z} f(X) dX \right) - \left( \int \frac{X - (I - L)}{X - (I - L)} f(X) dX + \frac{B}{Z} f(X) dX \right);$$

$$K = \left( \int \frac{B}{Z} Xf(X) dX - \int \frac{X - (I - L)}{X - (I - L)} f(X) dX \right). \quad (1)$$

Since $B = Z - (I - L)$, substituting and rearranging the terms of Equation 1, we get:

$$K = \int \left( 1 - \frac{(I - L)}{Z} \right) Xf(X) dX + \int \frac{(I - L)}{Z} f(X) dX;$$

$$K = \int \left( 1 - \frac{(I - L)}{Z} \right) Xf(X) dX + \int \frac{(I - L)}{Z} f(X) dX. \quad (2)$$

Since $(I - L) < Z$ and $X < Z$ over the interval $(I - L, Z)$ it follows that $K > 0$.

Proof of Proposition 1:

Follows immediately from Lemma 1, as higher priority always leads to greater investment. This higher investment is only optimal provided the borrower has enough good, positive NPV projects.

The result is intuitively appealing, as the higher priority on the DIP loan should induce the lender to finance a larger set of investment projects. Simple numerical examples illustrate the investment decisions made by the firm.

Case 1: Liquidation value is low but investment opportunities are good ($V_h > V_h^* > L$)

Consider a firm with pre-petition debt of face value B equal to 100, held by two lenders that equally share ($\phi = 0.5$). The firm has a liquidation value L equal to 80. One of the existing pre-petition lenders provides the DIP
financing. Further, let \( V^{**} = 90 \) and \( V^{*} = 110 \).\(^{134}\) If the DIP loan is at the same priority as the pre-petition loan, then the lowest level of NPV, for which a DIP lender would be willing to lend, can be estimated using Equation 4:

\[
(0.5)(110-80)=15.
\]

In this case, the firm would forego all projects with expected NPV between 0 and 15. Though, if the same firm were to obtain its DIP financing at a higher priority, then the cut-off level of NPV, above which the DIP lender would be willing to finance, is (using Equation 8):

\[
(0.5)(90-80)=5.
\]

Thus, the firm with higher priority DIP financing invests more than the firm with equal priority DIP financing. For a firm that has investment projects that are mostly positive NPV, higher priority of DIP financing is more likely to induce value increasing investments.

Case 2: Liquidation value as well as investment opportunities are mediocre \((V^{*} > L > V^{**})\)

Again, using the example described above, let us consider a firm for which \( V^{*} = 70 \), \( L = 60 \), and \( V^{**} = 50 \). If the firm receives a DIP loan with equal level priority to its pre-petition loans, it undertakes all projects with a NPV equal to:

\[
(0.5)(70-60)=5.
\]

While the cut off NPV project for a firm with a higher priority DIP loan is:

\[
(0.5)(50-60)=-5.
\]

The higher priority DIP financing results in the firm investing in negative NPV projects. In this case, equal priority of DIP loans results in better investment incentives.

Case 3: Liquidation value is high and investment opportunities are poor \((L > V^{*} > V^{**})\)

Again, using the example described above, let us consider a firm for which \( L = 50 \), \( V^{*} = 40 \), and \( V^{**} = 30 \). If the firm receives a DIP loan with equal

\(^{134}\) The value of \( V^{**} \) and \( V^{*} \) would also depend on distribution \( f(X) \) of cash flows as well as the liquidation value \( L \).
level priority to its pre-petition loans, it undertakes all projects with NPV equal to:

\[(0.5)(40-50)=-5.\]

While the cut off NPV project for a firm with a higher priority DIP loan is:

\[(0.5)(30-50)=-10.\]

Here the optimal decision is to liquidate the firm. If, on the other hand, DIP financing were to happen, equal priority would cause less value destruction.

**Proof of Proposition 2:**

Proposition 2 implies that cross-collaterlization would lead to even higher investment than a higher priority of DIP loan, i.e., minimum level of NPV required by a lender with a cross-collaterlized DIP loan is even lower than the cut-off NPV required by high-priority DIP lender. Thus, we need to show that \(V_b^H-V_b^C>0\). Since \(\phi B+(I-L)=Z-(1-\phi)B=T\). From Equations 7 and 11:

\[
V_b^H-V_b^C = \int_{\mathcal{L}} \left[ X - (I-L) \right] f(X) dX + \int_{\mathcal{S}} Bf(X) dX \]

\[
- \left( \int_{\mathcal{T}} Bf(X) dX + \int_{\mathcal{S}} \frac{X-(I+\phi B-L)}{1-\phi} f(X) dX \right); \]

\[
V_b^H-V_b^C = \int_{\mathcal{L}} \left[ X - (I-L) \right] f(X) dX - \int_{\mathcal{S}} \frac{X-(I+\phi B-L)}{1-\phi} f(X) dX. \quad (3)
\]

Let \(X-(I-L)=S\). Rearranging the right-hand side of Equation 3, we get:

\[
V_b^H-V_b^C = \int_{\mathcal{L}} Sf(X) dX + \int_{\mathcal{S}} \frac{Sf(X) dX - \int_{\mathcal{S}} \frac{X-(I+\phi B-L)}{1-\phi} f(X) dX}{1-\phi} \]

\[
= \int_{\mathcal{L}} Sf(X) dX + \frac{1}{1-\phi} \int_{\mathcal{S}} \left[ (1-\phi)S - S + \phi B \right] f(X) dX \]

\[
= \int_{\mathcal{L}} Sf(X) dX + \frac{\phi}{1-\phi} \int_{\mathcal{S}} [B+(I-L)-X] f(X) dX \]

\[
= \int_{\mathcal{L}} Sf(X) dX + \frac{\phi}{1-\phi} \int_{\mathcal{S}} [Z-X] f(X) dX \quad (4)
\]
Since $Z$ is always greater than $X$ for $\phi B + (I - L) < X < Z$, and $S > 0$ for $(I - L) < X < \phi B + (I - L)$, the right-hand side of Equation A-4 is always positive. Thus, $V^u_a - V^c_a > 0$.

**Proof of Proposition 3:**

We need to show that the minimum acceptable NPV, for a project to be acceptable to a new DIP lender, is higher than the one required by an existing lender. We derive this for the case when both the old and the new lender provide DIP loans at equal priority, and then, we derive the same result for higher priority DIP loans.

**Case 1: The new lender provides financing at the same priority as the pre-petition lender**

We first consider the case in which the DIP financing is provided by a new lender and at the same priority as the old loans. The incentive compatibility constraint for a new DIP lender is:

$$V^u_a - V^c_a > 0$$

$$\frac{\int_0^\infty I - L}{Z} X f(X) dX + \int_0^\infty ( X - B ) f(X) dX - ( I - L ) > 0$$

$$\frac{\int_0^\infty Z}{B} X f(X) dX + \int_0^\infty ( B ) f(X) dX - L$$

$$\frac{\int_0^\infty Z}{B} X f(X) dX + \int_0^\infty ( B ) f(X) dX - L$$

$$V^u_a$$ is as defined before in Equation 3.

Equation A-5 allows us to compare the investment policy of the firm, which obtained DIP financing from an existing lender, to the investment policy of the firm that obtained DIP from a new lender. The cut-off level of NPV for a project is $(1 - \phi) [V^a_u - L]$ for the firm with the DIP loan from its pre-petition lender (Equation 4). The lowest level of NPV project that is financed by a new DIP lender is $[V^c_B - L]$ (Equation A-5). Since $0 \leq \phi \leq 1$, the proof follows immediately.

**Case 2: The new lender provides financing at the higher priority than the pre-petition loans.**

In this case, a new lender provides the DIP financing, and the DIP financing carries higher priority than the old loans.
The new lender would finance provided that:

\[
\int_{\phi}^{1-\phi} X_f(x)dx + \int_{\phi}^{1-\phi} (I - L) f(x)dx + \int_{\phi}^{1-\phi} (X - B) f(x)dx - (I - L) > 0
\]

\[
\bar{X} - I > \int_{\phi}^{1-\phi} [X - (I - L)] f(x)dx + \int_{\phi}^{1-\phi} B f(x)dx - L;
\]

\[
\bar{X} - I > V_a^\mu - L.
\]

\(V_a^\mu\) is as defined in Equation 7.

Equation A-6 allows us to compare the investment policy of the firm, which obtained higher priority DIP financing from an existing lender, to the investment policy of the firm that obtained higher priority DIP financing from a new lender. The cut-off level NPV for a project is \((1-\phi)[V_a^\mu - L]\) for the firm with the DIP loan from its pre-petition lender (Equation 8). The lowest level of NPV project, financed by a new DIP lender, is \([V_a^\mu - L]\) (Equation 7).

Since \(0 \leq \phi \leq 1\), the proof follows immediately. The discussion of two cases shows that if the priority level of DIP loan by existing lender or new lender is assumed to be the same, the DIP financing by its existing pre-petition lender leads to a higher level of investment. In other words, the minimum acceptable NPV of a project is higher for a new DIP lender than the cut-off required by a DIP lender who already has loans outstanding. The intuition behind this is the fact that the new DIP lender receives the pay-off only from his DIP loan, while a pre-petition lender can achieve some additional pay-off if the value of the old loan increases. Again, for borrowers who face severe debt-overhang, but have good investment opportunities, allowing an existing lender to provide DIP financing would address the under-investment problem more effectively. If the borrower does not have many good investment projects though, the higher threshold required by a new DIP lender results in better investments.

**Proof of Proposition 4:**

Equation 4 provides the cut-off level of NPV required by an old lender willing to lend at equal priority, while Equation A-6 describes the same for a new DIP lender willing to lend at higher priority. The investment level would be higher for DIP loans from old lenders only if:

\[
(1-\phi)[V_a^\mu - L] < [V_a^\mu - L];
\]

\[
\phi > \frac{V_a^\mu - V_a^\mu}{V_a^\mu - L}.
\]
Again, the desirability of higher investment depends on the set of investment opportunities that the borrower has. If the firm needs to be provided with higher investment incentives, an equal priority DIP loan from an existing lender would achieve this outcome as long as the existing lender has a sufficiently large outstanding loan.

**Proof of Proposition 5:**

The intuition behind Proposition 5 is that if the firm has high liquidation value the senior creditors would suffer relatively smaller losses in case of liquidation and thus would only finance projects with relatively higher NPV. On the other hand, the junior debt-holders have little to gain from the liquidation and would finance a larger set of projects. However, if the firm has a very low liquidation value then the senior lenders would have incentives to lend as the investment would offer some probability of recovering their outstanding debt.

By assumption X: \( f(x) = 1/M \).

Substituting in Equation 16 we get:

\[
\frac{k_2 - k_1}{M} = \int_b^a \frac{B}{Z_i} \frac{X}{M} \, dx + \int_{b_2}^{a_2} (B-J) \, f(x) \, dx + \int_{b_1}^{a_1} (2B+I-L-X) \, f(x) \, dx - L
\]

(8)

Let us define the following:

\[
T_1 = \int_b^a \frac{B}{Z_i} \frac{X}{M} \, dx = \frac{BZ_i}{2M},
\]

(9)

\[
T_2 = \int_{b_2}^{a_2} (B-J) \, f(x) \, dx = \frac{(B-J)}{M} [M - Z_i],
\]

(10)

\[
T_3 = \int_{b_1}^{a_1} \frac{2B+I-L-X}{M} \, dx = \frac{(2B+I-L)}{M} [Z_i - Z_1] - \frac{(Z_i)^2 - (Z_1)^2}{2M}.
\]

(11)

The Equation 8 can be rewritten as:

\[
k_2 - k_1 = T_1 + T_2 + T_3 - L \Rightarrow
\]

\[
k_2 - k_1 = B \left( 1 - \frac{Z_i}{2M} \right) + f \left( \frac{Z_i + Z_1}{2M} - 1 \right) - L.
\]

(12)

If \( k_2 - k_1 < 0 \), we know that the firm will invest in a larger set of projects if the DIP financing is provided by the junior creditor. If \( k_2 - k_1 = 0 \), the investment decisions are unaffected by the choice of DIP lender. If \( k_2 - k_1 > 0 \), the investment is higher if DIP financing is from the senior creditor.
Since $J = T - B$ substitution for $J$ yields:

$$k_2 - k_1 = B \left( 1 - \frac{Z_1}{2M} \right) + (T - B) \left[ \frac{Z_1 + Z_2}{2M} - 1 \right] - L. \quad (13)$$

For $k_2 - k_1 = 0$, we get the critical liquidation value:

$$L^*_c(B) = \left[ B - \frac{I + B}{2M} \right] + (T - B) \frac{2I + T + B}{2M} \left[ 1 - \frac{B}{2M} \right] + \frac{T - B}{M}. \quad (14)$$

Further, $k_2 - k_1$ is a decreasing function of $L$ as shown below in Equation 15. Note that the right-hand side of Equation 15 is always negative under the assumption that $M > Z_2$.

$$\frac{\partial (k_2 - k_1)}{\partial L} = \frac{(B - 2J)}{2M} - 1. \quad (15)$$

It follows thus if $L$ is below the critical liquidation value ($k_2 - k_1 > 0$) then the DIP financing by the senior creditor leads to greater investment. On the other hand, if $L$ is above the critical liquidation value ($k_2 - k_1 < 0$) then the DIP financing by the junior creditor leads to higher investment.
Table 1
The Hierarchy of Financing Options for a Firm Operating Under Chapter 11

<table>
<thead>
<tr>
<th>Relevant Section of the Bankruptcy Code</th>
<th>Features of the Financing Arrangement</th>
<th>Security and Priority of the Post-petition debt</th>
<th>Legal Requirements</th>
</tr>
</thead>
<tbody>
<tr>
<td>§ 363(c)(2)</td>
<td>Use of Cash Collateral</td>
<td>Not Applicable</td>
<td>Approval of the bankruptcy court</td>
</tr>
<tr>
<td>§ 364(a)</td>
<td>Debt incurred in the ordinary course of business</td>
<td>Unsecured, Administrative Priority</td>
<td>No court approval required</td>
</tr>
<tr>
<td>§ 364(b)</td>
<td>Debt incurred for purposes other than ordinary course of business</td>
<td>Unsecured, Administrative Priority</td>
<td>Approval of the bankruptcy court, Notice and Hearing</td>
</tr>
<tr>
<td>§ 364(c)</td>
<td>Various subsections discussed below</td>
<td>Super-priority Debt</td>
<td>Debit has to show that it could not get financing under Sections 364(a) or 364(b)</td>
</tr>
<tr>
<td>§ 364(c)(1)</td>
<td></td>
<td>Unsecured, Senior Administrative Priority</td>
<td></td>
</tr>
<tr>
<td>§ 364(c)(2)</td>
<td></td>
<td>Lien on unencumbered assets, Senior Administrative Priority</td>
<td></td>
</tr>
<tr>
<td>§ 364(c)(3)</td>
<td></td>
<td>Junior lien on unencumbered assets, Senior Administrative Priority</td>
<td></td>
</tr>
<tr>
<td>§ 364(d)</td>
<td>Primed Debt</td>
<td>Secured by a senior or equal lien on assets already subject to lien or pledge</td>
<td>Same as Section 34(c) and show that existing holders of the security are adequately protected</td>
</tr>
<tr>
<td>§ 522(b)</td>
<td>Cross-collateralization</td>
<td>Collateral securing the pre-petition as well as post-petition debt</td>
<td>Court Approval (rarely given)</td>
</tr>
</tbody>
</table>
Table 2
Priorities of Major Creditor Classes for a Chapter 11 Firm

<table>
<thead>
<tr>
<th>Priority Rank</th>
<th>Claim and Relevant Priority Section</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>The primed lien DIP loans under § 364 (d).</td>
</tr>
<tr>
<td></td>
<td>Secured claim holders (up to the value of collateral) under § 506 (a).</td>
</tr>
<tr>
<td></td>
<td>Super-priority claims of the DIP lender under § 364 (c).</td>
</tr>
<tr>
<td></td>
<td>Administrative claims under § 507 (a) (1), § 503 (b), § 364 (a) and § 364 (b).</td>
</tr>
<tr>
<td></td>
<td>Involuntary gap creditor claims under § 507 (a) (2) and § 502 (f).</td>
</tr>
<tr>
<td></td>
<td>Unsecured claims (including the shortfall of collateral for secured claims) § 506 (a).</td>
</tr>
</tbody>
</table>